PD-480-CV-19

M.A./M.Sc. (4th Semester)

Examination, June-2021

MATHEMATICS

Paper-I

INTEGRATION THEORY & FUNCTIONAL ANALYSIS-II

Time : Three Hours]

[Maximum Marks : 80

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Fill in the Blanks of the following:-** 1x10=10

   (a) A normed linear space is a Banach space if and only if every................series is summable

   (b) Let N be a normed linear space and $x, y \in N$ Then $|\|x\| \|y\|| \leq$..................

   (c) Every complete subspace of a normed linear space is................

   (d) There exist a................real valued function whose fourier series diverges to zero

   (e) All norms are................on a finite dimensional space.

   (f) If S is non empty subset of a Hilbert space H then $S \cap S' \subset$....................

   (g) Set of all unitary operator form a........................

   (h) If T is positive operator on Hilbert space H then $I + T$ is.....................

   (i) An operator T on Hilbert space H is normal if and only if $\|T^{*}x\| =$............$\forall x \in H$.

   (j) Normed linear space is separable if it's..................is separable.

2. **Answer the following questions:-** 2x5=10

   (a) Define l_2 space.

   (b) State closed graph theorem.

   (c) State uniform bounded principle.

   (d) In Hilbert space H prove that $H^{\perp} = \{0\}$.

   (e) If T is normal operator on a Hilbert space H then prove that $\|T^{*}x\| = \|Tx\|$

Section-B

Answer the following questions:- 12x5=60

3. Prove that l^{∞} is Banach space.

   OR

   Let X be a normed space over the field K and let M be a closed subspace of X.
   $\|\cdot\|: \frac{X}{M} \to R$ defined by $\|x + M\|_1 = inf\{\|x + m\|: m \in M\}$
   Then $\left(\frac{X}{M}, \|\cdot\|_1\right)$ is a normed space further if X is a Bnanch space then $\frac{X}{M}$ is a Banach space

4. State and prove open mopping theorem.

   OR

   Let M be a closed linear subspace of a normed linear space N and let x_0 be a vector not in M. If d is distance from x_0 to M, then prove that there exist a function $f \in N^{*}$ such that $f(M) = \{0\}, f(x_0) = d \text{ and } \|f\| = 1$.

5. Let M be a normed linear space and A be a Banach space and let $T: M \to A$ is an onto isomorphism such that T and T^{-1} are continuous. Then prove that M is also a Banach space.

OR

Let $\{x_n\}$ be a weakly convergent sequence in a normed space X then prove that
(a) The weak limit of $\{x_n\}$ is unique.
(b) $\{\|x_n\|\}$ is a bounded sequence in R
(c) Every subsequence $\{x_n\}$ of converges weakly to the weak limit of $\{x_n\}$.

6. State and prove Riez representation theorm.

OR

Let M be a proper closed linear subspace of a Hilbert space H then prove that there exists a non zero vector z_0 in H such that $z_0 \perp M$.

7. Let T be an operator on a Hilbert space H. Then $\exists$ a unique operator T^* on H such that $(Tx, y) = (x, T^*y),\ \forall x, y \in H$

OR

Prove that adjoint operator $T \to T^*$ on $B(H)$ has following properties.
(i) $(T_1 + T_2)^* = T_1^* + T_2^*$
(ii) $(aT)^* = \bar{a}T^*$
(iii) $(T_1T_2)^* = T_2^*T_1^*$
(iv) $\|T^*\| = \|T\|$